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The continuity of multiplication for two topologies associated with a semifinite trace on von neumann algebra

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Abstract

Let M be a semifinite von Neumann algebra in a Hilbert space H and τ be a normal faithful semifinite trace on M. Let Mpr denote the set of all projections in M, e denote the unit of M, and ‖ · ‖ denote the C*-norm on M. The set of all τ-measurable operators M̃ with sum and product defined as the respective closures of the usual sum and product, is a *-algebra. The sets U(ε, δ)={x ε M̃: ‖xpk‖ ≤ ε and τ (e - p) ≤ δ for some p ε Mpr} ε>0; δ>0; form a base at 0 for a metrizable vector topology tτ on M̃, called the measure topology. Equipped with this topology, M̃ is a complete topological *-algebra. We will write xi τ→ x in case a net {xi} iεI ⊂ M̃ converges to x ε M̃ for the measure topology on M̃. By definition, a net {xi}iεI ⊂ M̃ converges τ-locally to x ε M̃ (notation: x i τl→ x) if xip τ→ xp for all p ε Mpr, τ(p) < ∞; and a net {xi} iεI ⊂ M̃ converges weak τ-locally to x ε M̃ (notation: xi wτl→ x) if xip τ→ pxp for all p ε Mpr, τ(p) < ∞.

Keywords

Compact operator, Convergence with respect to measure, Hilbert space, Measurable operator, Noncommutative integration, Semifinite trace, Topological algebra, Von Neumann algebra